Name of college – S. S. College J. Bad

Deptt – Mathematics

Topic – Alternating series and Test of its convergency.

class – B.Sc I (Sub)

Time –

Date 03-02-2021

By – Samrendra Kumar

Alternating series :→ In this series the terms are not necessarily positive but in which the signs alternate : This type is

$u_1 - u_2 + u_3 - u_4 + \cdots$ Where each

u_r is +ve.

Such series is called Alternating series.

Alternating Series Test :→

An infinite series in which the terms are alternately positive and negative is convergent if each term is numerically less than the preceding term at

$$\lim_{n\to\infty} u_n = 0$$

This test of convergency of infinite Alternating series is called Leibnitz Test :→

Proof :→ Let the series be denoted by

$u_1 - u_2 + u_3 - u_4 + u_5 - \cdots$

Where $u_1 > u_2 > u_3 > u_4 > u_5 > \cdots$

Now the given series can be written in the following two ways

$(u_1 - u_2) + (u_3 - u_4) + (u_5 - u_6) + \cdots$ — (I)

And $u_1 - [(u_2 - u_3) + (u_4 - u_5) + (u_6 - u_7) + \cdots]$ — (II)

~~my~~ Test are satisfied and so the series is convergent

As $u_{n-1} > u_n \quad \forall n \in N$

So From (I) we find Expression in each bracket is +ve and hence the sum of any number of terms of the Given Series is +ve.

From (II) we find that the Expression in each bracket being +ve, the sum of any ~~Term~~ No of Terms of the Given Series is always Less than u_1, a Finite quantity

Also $$S_{2n+1} = S_{2n} + u_{2n+1} \quad \forall n \in N \quad \text{——— (III)}$$

Also we are given that

$$\lim_{n \to \infty} U_n = 0 \quad \forall n \in N.$$

In particular

$$\lim_{n \to \infty} u_{2n+1} = 0$$

$\therefore$ From (III) we get

$$\lim_{n \to \infty} S_{2n+1} = \lim_{n \to \infty} S_{2n}$$

that is when $n \to \infty$, the sum of an even No of Terms and the sum of odd No of Terms do not differ.

Hence the Series is Not an oscillating One.

Hence the Given Series is Convergent

Ex:→ Test the convergency of the series

$$1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\cdots\cdots$$

Solution :→ In the Given Series we Find that

(I) the Terms are alternately positive and Negative

(II) the Terms are Continually decreasing

i.e $u_n < u_{n-1}$

(III) $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

Hence all three conditions of the alternating series Test are satisfied and so the Given Series is Convergent.

Ex:→

Test the series

$$\frac{2}{1^3}-\frac{3}{2^3}+\frac{4}{3^3}-\frac{5}{4^3}+\cdots\cdots$$

Solution:→ In the Given Series we Find that

(I) the Terms of Series are alternately positive and Negative.

(II) the Terms are Continually decreasing

and (III) $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{n+1}{n^3}$

$$= \lim_{n\to\infty} \frac{n(1+1/n)}{n^3}$$

$$= \lim_{n\to\infty} \frac{1+1/n}{n^2} = 0$$

Hence all the three Conditions of the alternating Series Test are satisfied and so the Series is Convergent

Ex:- Test the series
$$\sum_{n=1}^{\infty} \left[(-1)^n \frac{1}{2n-1} \right.$$

Solution → The given series is
$$\sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{2n-1}$$

putting $n = 1, 2, 3 \ldots\ldots$ we may write down the given series as

$$\frac{(-1)}{2\times1-1} + \frac{(-1)^2}{2\times2-1} + \frac{(-1)^3}{2\times3-1} + \frac{(-1)^4}{2\times4-1} + \ldots$$

$$\Rightarrow \frac{-1}{1} + \frac{1}{3} - \frac{1}{5} + \frac{1}{7}$$

For this series we find that -

(i) the terms are alternately +ve and -ve.

(ii) The terms are continually decreasing

and (iii) $\lim_{n\to\infty} |u_n| = \lim_{n\to\infty} \frac{1}{2n-1}$

$$= \lim_{n\to\infty} \frac{1}{n(2-1/n)} = \frac{0}{2-0} = 0$$

Hence all the conditions of the alternating series test are satisfied and so the series is convergent